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Recursive Fractal Image Coding Scheme with Feedback Structure*

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Abstract — Fractal image coding (FIC) has been studied for more than ten years and a lot of improvements on its fidelity performance have been proposed. In the paper, another improving scheme called recursive scheme is proposed. In the scheme, the coding results of previous range blocks are fed back to adjust the encoder parameters for the next range block. So the method can gradually reach the global optimization. Experimental results show that the method can improve the image quality about 0.5 dB higher than Fisher's quadtree scheme.

Key words — Fractal image coding, Image compression.

I. Introduction

The first automatic FIC scheme, which can compress monochrome images, was proposed by A. E. Jacquin in 1990[1]. The scheme modeled the original image as a fractal picture and made use of its block-wise self-similarity to search for the parameters of fractal transform. Since 1990, this theory and scheme have attracted a degree of attention and a lot of papers related to the subject have been published[2-13]. Y. Fisher modified the partition strategy and proposed a more practical scheme that can achieve better performance, and therefore the scheme becomes the most popular one[2].

However, as the research work of FIC goes deeper and deeper, some shortcomings are found, such as time-consuming encoding and unsatisfactory image quality. All the problems have motivated researchers to improve it. Synoptically, some researchers improve the image quality by adopting different partitions[2-5]. Some combine FIC with transform coding methods[6-8]. And others try to use nonlinear approximation in gray level[9,10]. In the paper, we will analyze the collage theorem, point out the relationships among the original image, the collage image and the decoded image, and then we will construct a feedback loop in the encoder and use the intermediate coding results to modify the encoding parameters.

Section II introduces the principle of FIC and analyses the problems, in order to solve the problem. Section III proposes a recursive coding scheme with feedback structure. Section IV presents the coding steps. At last the experimental results are presented in Section V.

II. Mathematical Foundation and Problems of FIC

Let (Ω, λ) denote a complete metric space. The original image X_{orig} is one element of the space. The fractal coding procedure of X_{orig} is trying to construct a transformation $T: \Omega \rightarrow \Omega$, which satisfies the following conditions:

(i) For any two elements $x_1, x_2 \in \Omega$,

$$\lambda(T(x_1), T(x_2)) \leq s \cdot \lambda(x_1, x_2) \tag{1}$$

where $0 \leq s < 1$;

(ii) $$T(X_{orig}) \approx X_{orig} \tag{2}$$

From condition (i), we know that T is a contractive transformation, and s is its contractivity factor. For a contractive transform, there exists Collage Theorem.

Collage Theorem Let (Ω, λ) be a complete metric space and $T: \Omega \rightarrow \Omega$ be a contractive transform with contractivity factor s, then there exists a unique fixed point $x_{fix} \in \Omega$ such that for any point $x \in \Omega$

$$x_{fix} = T(x_{fix}) = \lim_{n\to\infty} T^{on}(x) \tag{3}$$

$$\lambda(x, x_{fix}) \leq \frac{1}{1-s}\lambda(x, T(x)) \tag{4}$$

Collage Theorem tells us that T has a unique fixed point and the fixed point can be found by iterative transformation of T on any element. Let $x = X_{orig}$ then Eq.(4) becomes:

$$\lambda(X_{orig}, x_{fix}) \leq \frac{1}{1-s}\lambda(X_{orig}, T(X_{orig})) \tag{5}$$

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From condition (ii), we know that X_{orig} is an approximate fixed point of T. That is to say, $\lambda(X_{orig}, T(X_{orig}))$ is very small. $T(X_{orig})$ is usually called the collage of X_{orig}. Collage Theorem guarantees that if there is a transform T that makes $T(X_{orig})$ close enough with X_{orig}, then the fixed point x_{fix} will be also close to X_{orig}. In FIC, x_{fix} is actually the decoded image, since it can be constructed by applying T on any initial image X_0 iteratively. Usually T can be stored compactly, and it is called the compressed data of X_{orig}. Therefore, X_{orig} is compressed.

Eq.(5) means that if the difference between the original image and its collage is small enough, then the difference between the decoded image and the original is also small. It is a sufficient condition, however, not a necessary condition. So the difference between the original and the decoded image cannot be directly affected by the difference between the original and its collage. Thus in the encoding procedure, minimizing the difference between the original image and its collage usually does not result in the minimizing of the difference between the decoding and the original. Fig.1 illustrates the collage images and decoded images of two FIC experiments.

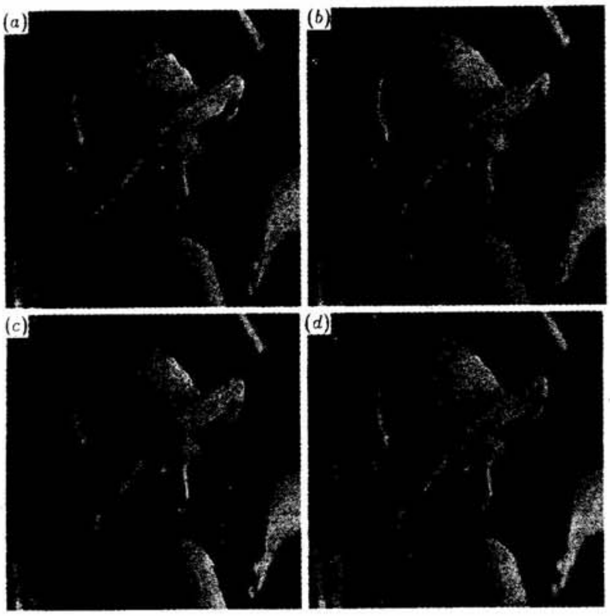

Fig. 1. (a) The collage image of FIC A, PSNR=33.38dB; (b) The decoded image of FIC A, PSNR=28.92dB; (c) The collage image of FIC B, PSNR=32.15dB; (d)The decoded image of FIC B, PSNR=31.51dB

Comparing the two FICs, we know that the quality of the collage image of FIC 1 is better than that of FIC 2, however, the decoded image of FIC 1 is worse than FIC 2.

Several researchers have noticed this problem and have proposed alternative solutions[11-13]. In these works, the common idea is using an iterative coding scheme, i.e. the original image is encoded and decoded for several times. In each iteration, fractal transforms are got using conventional FIC, and then the domain pool is updated for the next iteration with the decoded image. In each iteration, a conventional fractal image coding and decoding are needed, and the iteration number is unknown in advance. Therefore, the main drawback of these schemes is their heavy computation burden.

In order to improve the performance without significantly increasing the computation burden, a novel coding method is proposed in the paper.

III. Recursive Coding Scheme with Feedback

In fact, in image compression, we only concern the difference between the original and the decoded instead of the difference between the original and its collage. So in FIC, if we can minimize the difference between the decoded image and the original in the encoding procedure, the compression fidelity must be improved. However, this is impossible, since only after the whole image is encoded, the transforms are obtained and only after the transforms are obtained, the decoded image can be got.

Fortunately, in FIC scheme, the original image is encoded one range block after another, and for every range block, we obtain an affine transform. So in the encoding procedure, we can get an approximate decoded image (we call it domain image later) by iteratively transforming the original image using affine transforms got, then we use the domain image to encode the next range block. The recursive-coding diagram is shown in Fig.2.

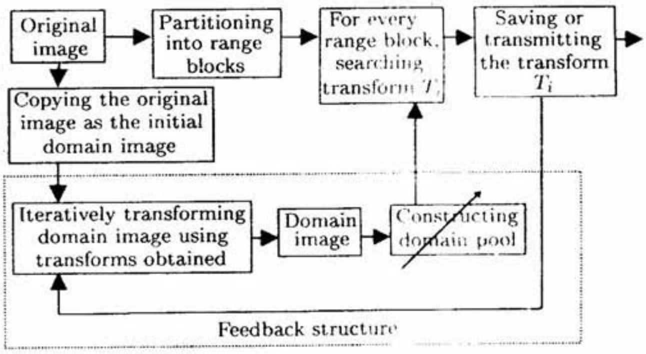

Fig. 2. The recursive coding scheme for FIC

In the above diagram, the original image is partitioned into range blocks, meanwhile, it is copied as initial domain image. For a range block, its domain pool is constructed from the domain image, and we search for an affine transform and its matching domain block in the domain pool, when the matching domain and transform are found, we save or transmit its parameters. Meanwhile we iteratively transform the domain image for several times using the transforms we have obtained, and therefore we get a new domain image and it will be used for next range encoding. The range blocks are encoded one by one along with the domain image changing.

IV. Coding Steps

The concise coding steps of our scheme are as follows:

(1) For an original image X_{orig} to be encoded, copy X_{orig} as initial domain image X_{dom};

(2) Quadtree partition X_{orig} into range blocks similar to Fisher's scheme[2].

(3) Construct domain blocks pool D from X_{dom};

(4) For a range block, try to find a domain block from domain pool D and calculate optimal affine parameters to minimize the rms (root mean square) error between the range block and the domain block. If rms error is less than a pre-selected threshold TOL, the matching domain block and affine transform are found, then the range block is encoded. If all the domains in D can not satisfy the thresholds, split the range into four quadtree ranges and go to step (3);

(5) If the range block is encoded in the above step, update the domain image X_{dom} by transforming X_{dom} using all the transforms achieved for several times, go to step (3);

(6) When every range block is encoded, the whole image is therefore encoded.

V. Experimental Results

This experiment is done to compare the performance with Fisher's quadtree scheme[2]. $512 \times 512 \times 8$ "Zelda" is the test image. The peak-to-peak signal to noise ratio (PSNR) and compression ratios (CR) of the two schemes vs. different tolerances are shown in Table 1.

Table 1. PSNR and CR vs. different tolerances

TOL		4	8	12	16	20	24	28
Scheme proposed	CR	7.16	32.78	46	84.26	137.68	193.36	253.03
	PSNR (dB)	36.65	33.01	30.3	28.34	27.25	26.46	26.17
Fisher's scheme	CR	7.16	32.62	45.93	84.10	140.26	204.16	255.75
	PSNR (dB)	36.11	32.60	29.73	27.95	26.61	25.81	25.47

Analyzing the above table, we know that at different tolerances, the compression ratios of the two methods are almost the same, the PSNR of our method is about 0.5 dB higher.

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